Intro Video: Section 5.5 (part 2) More complicated integration by substitution

Math F251X: Calculus I

Example: $\displaystyle\int \frac{2^t}{2^t+3}\,dt$

Let $u = 2^t$. Then $\frac{du}{dt} = 2^t \ln(2) \Rightarrow \frac{du}{2^t \ln(2)} = dt$

So $\displaystyle\int \frac{2^t}{2^t+3}\,dt = \int \frac{\cancel{2^t}}{u+3}\left(\frac{du}{\cancel{2^t}\ln(2)}\right) = \int \frac{1}{\ln(2)(\underline{u+3})}\,du$ ☹

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Let $u = 2^t + 3$. Then $\frac{du}{dt} = 2^t \ln(2) \Rightarrow \frac{du}{2^t \ln(2)} = dt$

So $\displaystyle\int \frac{2^t}{2^t+3}\,dt = \int \frac{\cancel{2^t}}{u}\left(\frac{du}{\cancel{2^t}\ln(2)}\right) = \frac{1}{\ln(2)}\int \frac{1}{u}\,du$

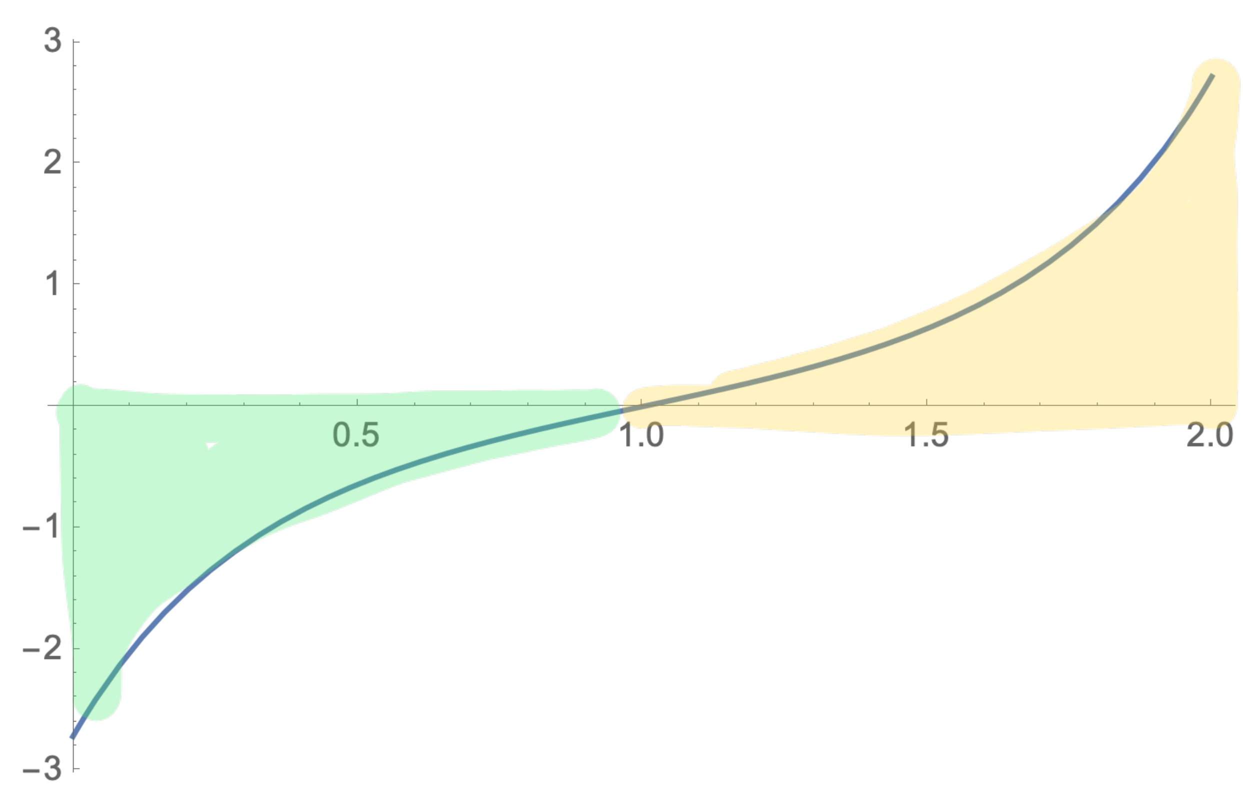
$\displaystyle = \frac{1}{\ln(2)} \ln|u| + C = \frac{1}{\ln(2)} \ln|2^t+3| + C$

Example: $\displaystyle\int_0^2 (x-1)\, e^{\boxed{(x-1)^2}}\, dx$

Let $u = (x-1)^2 \Rightarrow \dfrac{du}{dx} = 2(x-1) \Rightarrow \dfrac{du}{2(x-1)} = dx$

If $x=0$, $u = (0-1)^2 = 1$. If $x=2$, $u=(2-1)^2 = 1$

So $\displaystyle\int_0^2 (x-1)\, e^{(x-1)^2} dx = \int_1^1 \cancel{(x-1)}\, e^u \frac{du}{2\cancel{(x-1)}} = 0$



Plot of $(x-1)\, e^{(x-1)^2}$ on interval $[0,2]$

Example: Oil leaks from a tanker at a rate of

$$r(t) = 100e^{-0.01t}$$

liters/minute. How much oil leaks out during the first hour, if $A(t)$ is the amount of leaked oil and $A(0)=0$?

Know $A(1) - A(0) = \int_0^1 r(t)\,dt = \int_0^1 100e^{-0.01t}\,dt$

Let $u = -0.01t \Rightarrow \frac{du}{-0.01} = dt$. Note $t=0 \Rightarrow u=0$, and $t=1 \Rightarrow u = -0.01 = -\frac{1}{100}$

$$A(1) = \int_{u=0}^{u=-1/100} 100e^u \cdot \frac{du}{-0.01} = 100(-100)\int_0^{-1/100} e^u\,du = -10000\left[e^{-1/100} - e^0\right]$$

$$= -10000\left[\frac{1}{e^{1/100}} - 1\right] \approx 99.5 \text{ liters}$$

Example: $\displaystyle\int \frac{(\arctan(x))^2}{1+x^2}\,dx$

Let $u = \arctan(x)$. Then $\dfrac{du}{dx} = \dfrac{1}{1+x^2} \Rightarrow du(1+x^2) = dx$

So $\displaystyle\int \frac{(\arctan(x))^2}{1+x^2}\,dx = \int \frac{u^2}{\cancel{(1+x^2)}} \cdot \cancel{(1+x^2)}\,du = \int u^2\,du$

$\displaystyle = \frac{u^3}{3} + C = \frac{(\arctan(x))^3}{3} + C$

Example: $\int x^2 \sqrt{2+x}\, dx$

Let $u = x^2 \Rightarrow \frac{du}{dx} = 2x \Rightarrow \frac{du}{2x} = dx$

So $\int x^2 \sqrt{2+x}\, dx = \int u \sqrt{2+x}\, \frac{du}{2x} = \int \frac{u\sqrt{2+x}}{2x}\, du$

$= \int u \sqrt{\frac{2+x}{4x^2}}\, du = \int u \sqrt{\frac{1}{2x^2} + \frac{1}{4x}}\, du$ $\boxed{\text{No good!}}$

Let $u = 2+x$. Then $du = dx$ and $x = u-2$.

So $\int x^2 \sqrt{2+x}\, dx = \int (u-2)^2 \sqrt{u}\, du = \int (u^2 - 4u + 4) u^{1/2}\, du$

$= \int u^{5/2} - 4u^{3/2} + 4u^{1/2}\, du = \frac{2}{7} u^{7/2} - 4 \cdot \frac{2}{3} u^{3/2} + 4 \cdot 2u^{1/2} + C$

$= \frac{2}{7}(2+x)^{7/2} - \frac{8}{3}(2+x)^{3/2} + 8(2+x)^{1/2} + C$

Example: $\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx$

Let $u = \cos(x)$. Then $\frac{du}{dx} = -\sin(x) \Rightarrow \frac{du}{-\sin(x)} = dx$

So $\int \frac{\sin(x)}{\cos(x)}\,dx = \int \frac{\cancel{\sin(x)}}{u}\left(\frac{du}{-\cancel{\sin(x)}}\right) = -\int \frac{1}{u}\,du$

$= -\ln|u| + C = -\ln|\cos(x)| + C$

Note $\frac{d}{dx}\left(-\ln|\cos(x)| + c\right) = -\frac{1}{\cos(x)}(-\sin(x)) = \frac{\sin(x)}{\cos(x)}$